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# Separation of boron isotopes in optimal cascade of uniflow gas centrifuges

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**Abstract.** Was considered a problem of an optimization of concurrent gas centrifuges for separation of Boron isotopes in form of trifluoride  $\text{BF}_3$ . As the criteria was used a minimum of total number of gas centrifuges upon the given external parameters of the cascades' scheme. The method is based on the analytical relationships for the flows of stages, received under approximating minimization of the total feed flow. Conducted cascade calculations showed that it is possible to obtain  $\text{BF}_3$  with enrichment up to 99.9 % of  $^{10}\text{B}$  in the selection and up to 0.1 % in the waste, which is equivalent to 99.9 % of  $^{11}\text{B}$ .

## 1. Introduction

Boron at 80-95 % concentration of  $^{10}\text{B}$  is used, in nuclear reactor industry, mainly as a component of the control rods of the reactor control and safety system.  $^{11}\text{B}$  heavy isotope as opposed to the light one has substantially smaller value of thermal-neutron capture cross-section. This is used for production of construction materials which are heat-resisting and "transparent" with respect to neutron [1–2].

Production of stable isotopes is strongly related with usage of gas centrifuges [3–4]. Equipment used for setting a counter-current flow in gas centrifuges (separator-circulator, orifice plates, water cooling, etc.) not only sophisticates the construction and reduces the centrifuge reliability but also limits the permissible feed, selection and waste flows. In this connection the interest is presented by the simple concurrent centrifuges, optimized for the commercial enrichment of Boron trifluoride, which contain no auxiliary equipment to sophisticate the construction.

In the present paper were conducted optimization calculations of cascades for Boron isotopes separation. The purpose of optimization is a search of cascade internal parameters upon the given external ones that minimize the total amount of gas centrifuges.

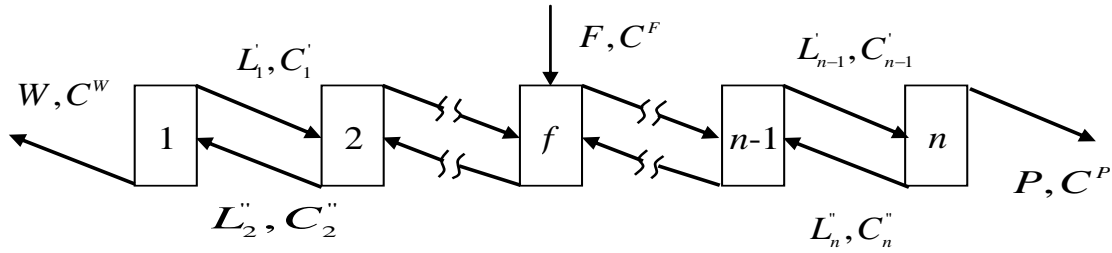
## 2. Cascade parameters and equations

To the external parameters of ordinary (triple-flow) centrifugal cascade, working in accordance with the symmetric counter-current scheme (figure 1), belong  $F$ ,  $P$ ,  $W$  – feed, selection and waste flows,  $C^F$ ,  $C^P$ ,  $C^W$  – concentrations of light isotope in feed, selection and waste. The external parameters are linked to the equations of matter and light isotope balance

$$\begin{cases} F = P + W \\ FC^F = PC^P + WC^W \end{cases}$$

Consequently only four parameters out of six are independent.





**Figure 1.** The scheme of counter-current symmetrical cascade.

To the internal parameters belong:  $L_i, L'_i, L''_i$  – feed, selection and waste flows of  $i$ - stage ( $i = \overline{1, n}$ );  $C_i, C'_i, C''_i$  – feed, selection and waste concentrations of  $i$ - stage;  $n$  – number of stages in a cascade;  $f$  – stage number of cascade power feed. These parameters are linked to  $2n$  equations of matter and light isotope balance, written for each stage. The interconnection stage scheme also generates  $2n$  balance equations for the interstage flows. Additionally to these relations a consideration must be given to the 4 terminal conditions, linking the external and internal parameters, and separating characteristics of gas centrifuges. The total separation factor of the  $q_i$  stage is

$$q_i = \frac{C'_i / (1 - C'_i)}{C''_i / (1 - C''_i)}, \quad i = \overline{1, n},$$

$$q_i = q_i(l_i, \theta_i), \quad i = \overline{1, n},$$

where  $q_i = q_i(l_i, \theta_i)$  – dependence of  $q_i$  on centrifuge feed flow  $l_i$  and split ratio  $\theta_i$ . There parameters are equal to

$$\theta_i = L'_i / L_i, \quad l_i = L_i / N_i,$$

where  $N_i$  – number of gas centrifuges in the  $i$ - stage.

### 3. Optimization criterion and equations.

Manufacturing expenses are in proportion to the total number of centrifuges. Consequently, as an optimization criterion it is advisable to take the minimum  $\sum N_i$ . Quantity of independent parameters of ordinary cascade of gas centrifuges is  $2n + 4$ . In case when 4 external parameters and  $n$  splitting ratio of stages are set, there are left  $n$  internal variables, which can be optimized in accordance with the accepted criterion. As such variables it is convenient to take the  $n$  number of stages,  $f$  stage number of feed flows and waste flows  $L''_i, i = \overline{3, n}$ .

Preliminary calculations of Boron isotope separation in cascades showed that the close approximation for the preferable  $L''_i$  upon the given parameters  $n$  and  $f$  is determined by the set of equations of total flow optimization [5]

$$(L''_i)^2 - \tau_i L''_i - \varphi_{i-1} (L''_{i-1}) = 0, \quad i = \overline{3, n},$$

where

$$\varphi_{i-1} = \frac{(\tau_i C'_{i-1} - \tau''_i) C''_{i-1} (1 - C''_{i-1})}{(\tau_{i-1} C'_{i-2} - \tau''_{i-1}) C'_{i-1} (1 - C'_{i-1})}. \quad (1)$$

Transfer matter and specific isotope flows towards the cascade waste  $\tau_i, \tau''_i$  are

$$\tau_i = \begin{cases} W, & i = \overline{1, f} \\ -P, & i = \overline{f+1, n} \end{cases}$$

$$\tau_i^u = \begin{cases} C^W W, & i = \overline{1, f} \\ -C^P P, & i = \overline{f+1, n} \end{cases}$$

Concentrations  $C'_{i-1}$ ,  $C'_{i-2}$ ,  $C''_{i-1}$  in (1) are deduced from  $L''_2, L''_3, \dots, L''_{i-1}$ . For this reason the optimization task resolves itself to a direct search of  $L''_2$  and a calculation of the remaining flows as follows

$$L''_i = \frac{\tau_i}{2} + \sqrt{\left(\frac{\tau_i}{2}\right)^2 + \varphi_{i-1} (L''_i)^2}, \quad i = \overline{3, n}.$$

The search of  $L''_2$  is carried out for as long as the condition  $C'_n = C^P$  is true.

#### 4. Concurrent centrifuge with the center body

Diffusion problem solving for the concurrent centrifuge with cylindrical central body with the radius  $r_0$  leads to the splitting ratio [6]:

$$q = 1 + \varepsilon(r_0, r_1, a) [1 - \exp(-\beta H)], \quad (2)$$

where  $a$  – rotor radius,  $\beta^{-1}$  – parameter, revealing the typical scale of axial concentration change,  $H$  – rotor length.

The separating capacity is defined as follows

$$\delta U = 2\pi\rho DH \frac{\varepsilon^2(r_0, r_1, a) [1 - \exp(-\beta H)]^2}{\ln(\xi_1^{-2}) \beta H},$$

where the extreme obtainable ratio of enrichment is

$$\varepsilon = \Delta A \frac{1 - \xi_1^2 - \xi_0^4 (1 - \xi_1^{-2}) - 4\xi_0^2 \ln(\xi_1^{-1})}{(1 - \xi_0^2)^2},$$

where  $\rho$  – mixture density,  $D$  – diffusion constant,  $\Delta A = \Delta M V^2 / 2RT$ ,  $\Delta M$  – difference between molar weight of the heavy and light components,  $V$  – surface speed of the wall,  $T$  – gas temperature,  $R$  – absolute gas constant,  $\xi_0 = r_0/a$ ,  $\xi_1 = r_1/a$  – non-dimensional radial coordinates,

$$\beta = \frac{2\pi\rho D}{\theta(1-\theta)l \ln(\xi_1^{-1})}. \quad (3)$$

Evaluation of the mixture separation  $BF_3$  in the Iguasu model centrifuge with the center body ( $r_0 = 0.01$  m,  $r_1 = 0.015$  m,  $a = 0.06$  m,  $H = 0.48$  m,  $V = 600$  m/s,  $T = 320$  K) gives the following  $A = 4.535$ ,  $\Delta A = 0.068$ ,  $\theta = 0.5$ ,  $q = 1.0407$ ,  $l_{opt} = 0.152$  g/sec,  $\varepsilon_s = 0.057$ . The optimal flow  $l_{opt}$  corresponds to the maximum of the separating capacity.

#### 5. Calculation of cascade parameters

In the cascade calculation were set independent external parameters  $F$ ,  $C^F$ ,  $C^P$ ,  $C^W$ .  $P$  and  $W$  flows were found from the cascade balance equations. Full splitting ratio  $q_i$  is considered to be identical for all the stages. The optimal number of the stages  $n$  and stage number of feed flow  $f$  were defined by the direct search and  $L''_i$ ,  $i = \overline{2, n}$ , flows were calculated in accordance with the optimization equations. The starting values of the feed flow point and quantity of stages were calculated using the formulas for the ideal cascade with the symmetrical stages:

$$f = \frac{\ln(\frac{R^F}{R^W})}{\ln \alpha}, \quad n = \frac{\ln(\frac{R^P}{R^F})}{\ln \alpha} + p - 1,$$

where  $R = C/(1 - C)$  – relative concentration,  $\alpha = \sqrt{q}$  – splitting ratio in selection.

Cascade parameters were calculated for each stage, starting from the waste cascade, as follows:

1st stage:

$$L''_1 = W, \quad C''_1 = C^W,$$

$$L'_2 = L''_2 - \tau_2, \quad L_1 = L'_1 + L''_1,$$

$$C'_1 = \frac{q_1 C''_1}{1 + (q_1 - 1)C''_1}, \quad \theta_1 = \frac{L'_1}{L_1}.$$

2, ..., (n - 1) stages:

$$L''_i = \frac{\tau_i}{2} + \sqrt{\left(\frac{\tau_i}{2}\right)^2 + \phi_{i-1}(L''_{i-1})^2}, \quad i = \overline{3, n},$$

$$C''_i = C'_{i-1} - \frac{\tau_i C'_{i-1} - \tau''_i}{L''_{i-1}}, \quad C'_i = \frac{q_i C''_i}{1 + (q_i - 1)C''_i},$$

$$L'_i = L''_{i-1} - \tau_{i+1}, \quad L_i = L'_i + L''_i, \quad \theta = \frac{L'_i}{L_i}.$$

n-th stage:

$$L''_n = \frac{\tau_n}{2} + \sqrt{\left(\frac{\tau_n}{2}\right)^2 + \phi_{n-1}(L''_{n-1})^2},$$

$$C''_n = C'_{n-1} - \frac{\tau_n C'_{n-1} - \tau''_n}{L''_{n-1}}, \quad C'_n = \frac{q_n C''_n}{1 + (q_n - 1)C''_n},$$

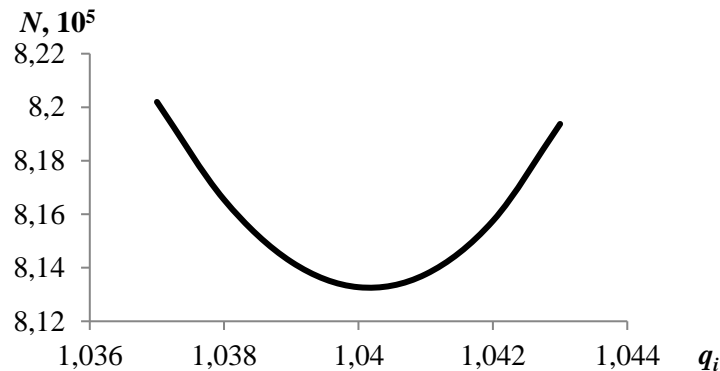
$$L'_n = P, \quad L_n = L'_n + L''_n, \quad \theta_n = \frac{L'_n}{L_n}.$$

Number of gas centrifuges in the stages is  $N_i = L_i/l_i$ . From this in accordance with the equations (2), (3),  $N_i$  was determined as follows

$$N_i = \frac{L_i \ln(1 - \frac{q_i - 1}{\epsilon_s}) \theta_i (1 - \theta_i) \ln(\xi_1)}{2\pi\rho DH}.$$

## 6. The optimal cascade for the enrichment by $^{10}\text{B}$

As the initial data for the calculations of the first ordinary cascade were set:  $F = 10$  g/sec,  $C^W = 5$  %,  $C^F = 19.8$  %,  $C^P = 99.9$  %. Splitting ratios were set identical for each stage. A direct search was made and a total number of centrifuges  $N$  was calculated to determine an optimal  $q_i$ . The calculation data is given on the figure 1. These data indicate that the extremum of the function  $N(q_i)$  corresponds to  $q_i = 1.040$ . This value approximately corresponds to the maximum of separating capacity of gas centrifuge.

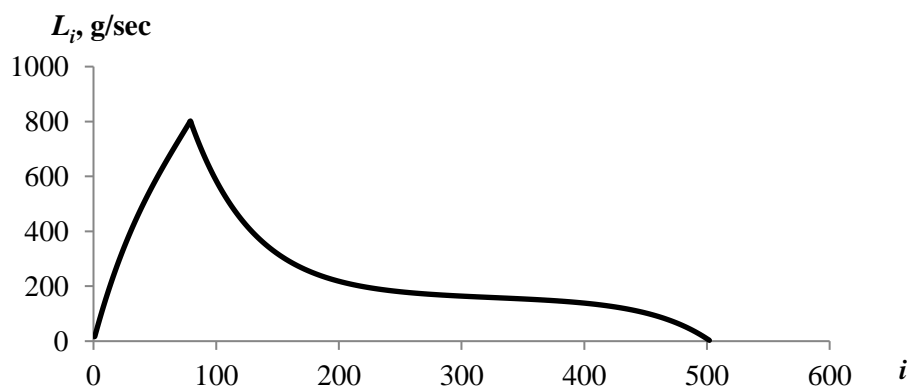


**Figure 2.** Dependence of the total number of centrifuges on splitting ratio.

In figure 3 is given the distribution of the feed flow through the stages. It has a usual long-tailed form towards the strong concentration of  $^{10}\text{B}$ . The basic parameters of the first cascade are shown in the table 1.  $^{10}\text{B}$  extraction into selection is 15.6 %.

**Table 1.** Parameters of the optimal ordinary cascades.

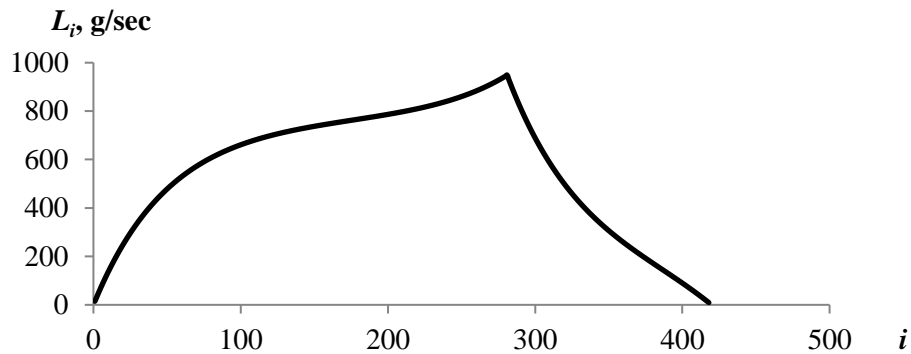
$N_0$	$F$ , g/sec	$P$ , g/sec	$W$ , g/sec	$C^F$ , %	$C^P$ , %	$C^W$ , %	$n$	$f$	$\Sigma N$
<b>1st cascade</b>	10	1.56	8.44	19.8	99.9	5	502	79	813273
<b>2nd cascade</b>	10	2.19	7.81	19.8	79	0.1	419	281	1488638
<b>3rd cascade</b>	10	1.97	8.03	19.8	99.9	0.1	704	281	1993926



**Figure 3.** Distribution of feed flow  $L_i$  through the stages of the first cascade.

## 7. Optimal cascade for enrichment by $^{11}\text{B}$

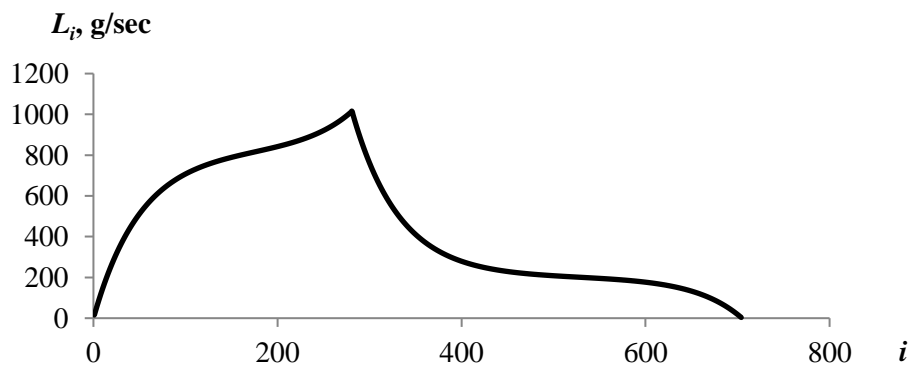
For the second ordinary cascade with the enrichment by  $^{11}\text{B}$  were set:  $F = 10$  g/sec,  $C^W = 0.1$  %,  $C^F = 19.8$  %. The other parameters were calculated based on the balance equations so as the extraction of  $^{11}\text{B}$  from feed into the waste was approximately 75 %. The calculations were carried out for the optimal  $q_i = 1.040$ . In figure 4 is given the graph of changes of the feed flow through the stages. By contrast to the first cascade it is described by the bigger flows in the waste part. The total number of gas centrifuges is 1.8 times greater (table 1).



**Figure 4.** Distribution of the feed flow  $L_i$  through the stages of the second cascade.

### 8. Optimal cascade for the enrichment by $^{10}\text{B}$ and $^{11}\text{B}$

For the simultaneous enrichment by both isotopes, in the third ordinary cascade were set:  $F = 10$  g/sec,  $C^W = 0.1$  % (99.9 % of  $^{11}\text{B}$ ),  $C^F = 19.8$  %,  $C^P = 99.9$  %. The calculations were carried out for the optimal  $q_i = 1.040$ . Number of stages, number of gas centrifuges (table 1) and feed flows (figure 5) were increased as compared with the first two cascades.



**Figure 5.** Distribution of the feed flow  $L_i$  through the stages of the third cascade.

### 9. Optimal double cascade for the enrichment by $^{10}\text{B}$ and $^{11}\text{B}$

As compared with the third ordinary cascade was considered an optimization of two-stage scheme, in which the waste of the first cascade is the feed for the second one. Use of two cascades scheme for the simultaneous enrichment by both isotopes can be efficient when it is impossible to build a long cascade. Rather can be used an equivalent made of two shorter cascades.

**Table 2.** Characteristics of the effective two-cascade scheme.

$N_0$	$F$ , g/sec	$P$ , g/sec	$W$ , g/sec	$C^F$ , %	$C^P$ , %	$C^W$ , %	$n$	$f$	$\sum N$
<b>First cascade</b>	12.66	1.97	10.68	19.8	99.9	5	502	79	1029378
<b>Second cascade</b>	10.68	2.66	8.03	5	19.8	0.1	280	202	964543

The external parameters of the first cascade selection and the second cascade waste were in conformity with the parameters of enrichment by  $^{10}\text{B}$  and  $^{11}\text{B}$  of the ordinary cascade (Table 2). The calculations have shown a match of both compared schemes. In case of total number of gas centrifuges is about two millions, the difference in schemes is five units.

## 10. Conclusion

The method of number-analytic optimization of cascades of concurrent gas centrifuges for Boron isotopes separation was worked out. Was shown the availability of  $\text{BF}_3$  with high enrichment by both  $^{10}\text{B}$  and  $^{11}\text{B}$  in one cascade and in scheme of two cascades.

## References

- [1] Shmelev A N, Smirnov A Yu, Bonarev A K, Borisevich V D, Kulikov G G and Sulaberidze G A 2016 *Theor. Found. Chem. Eng.* **50**(6) 1049
- [2] Chibak A F and Polevoy A S 2005 Isotopes in reactor engineering *Isotopes: properties, production, application* vol 2, ed Baranov V Yu (Moscow: Fizmatlit) pp 192–232
- [3] Orlov A A, Ushakov A A and Sovach V P 2019 *Theor. Found. Chem. Eng.* **53**(2) 193
- [4] Smirnov A Yu and Sulaberidze, G A 2014 *Theor. Found. Chem. Eng.* **48**(5) 629
- [5] Palkin, V A 1997 *At. Energy* **82**(4) 288
- [6] Tokmantsev V I and Palkin V A 2017 *At. Energy* **123**(1) 49